Lecture 24

Dynamic Programming: Rod Cutting (contd.), LCS

Source: Introduction to Algorithms, CLRS

When to Use Dynamic Programming?

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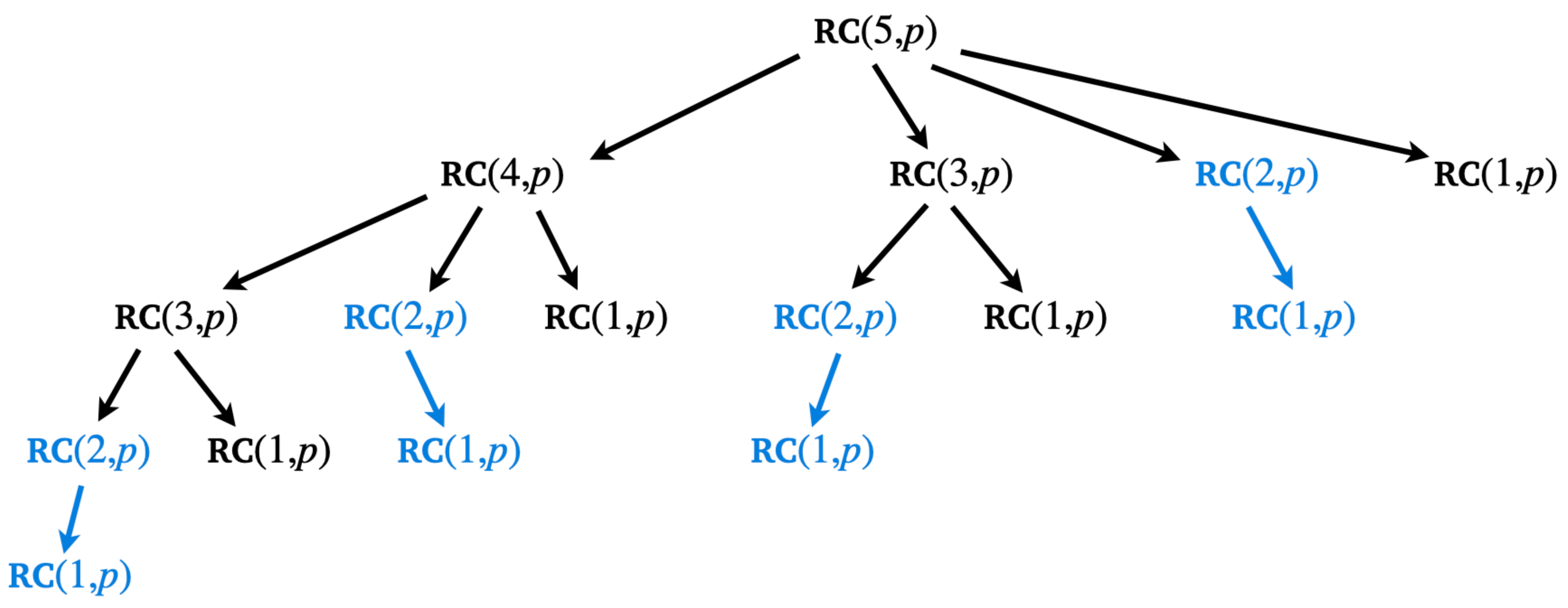

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Solving a problem using dynamic programming usually takes three steps:

- Find the optimal substructure.
- Recursively define the value of optimal solution.
- Compute the value of the optimal solution using an array.

Bottom-Up Dynamic Programming

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Time-Complexity: $O(n^2)$ due to loops of line 2 and 4.

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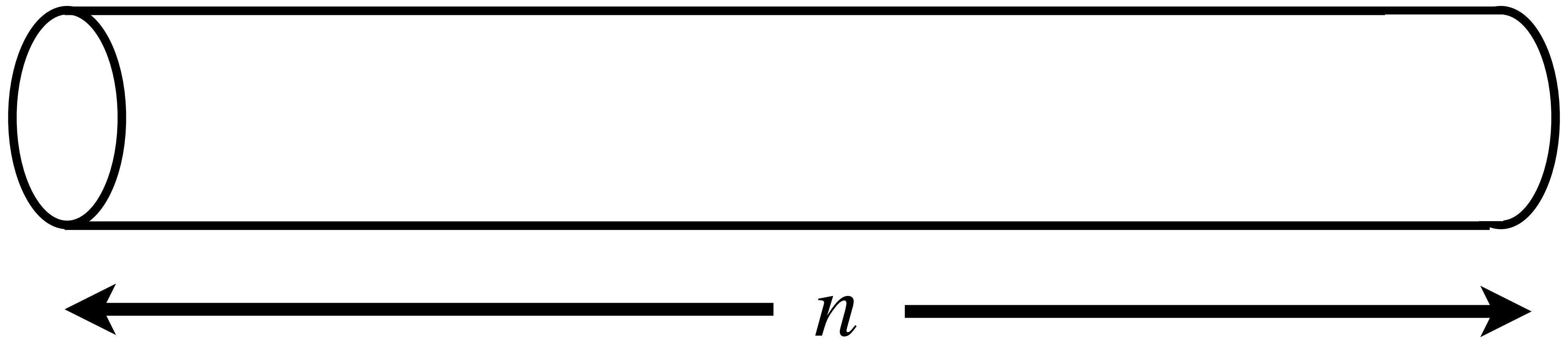

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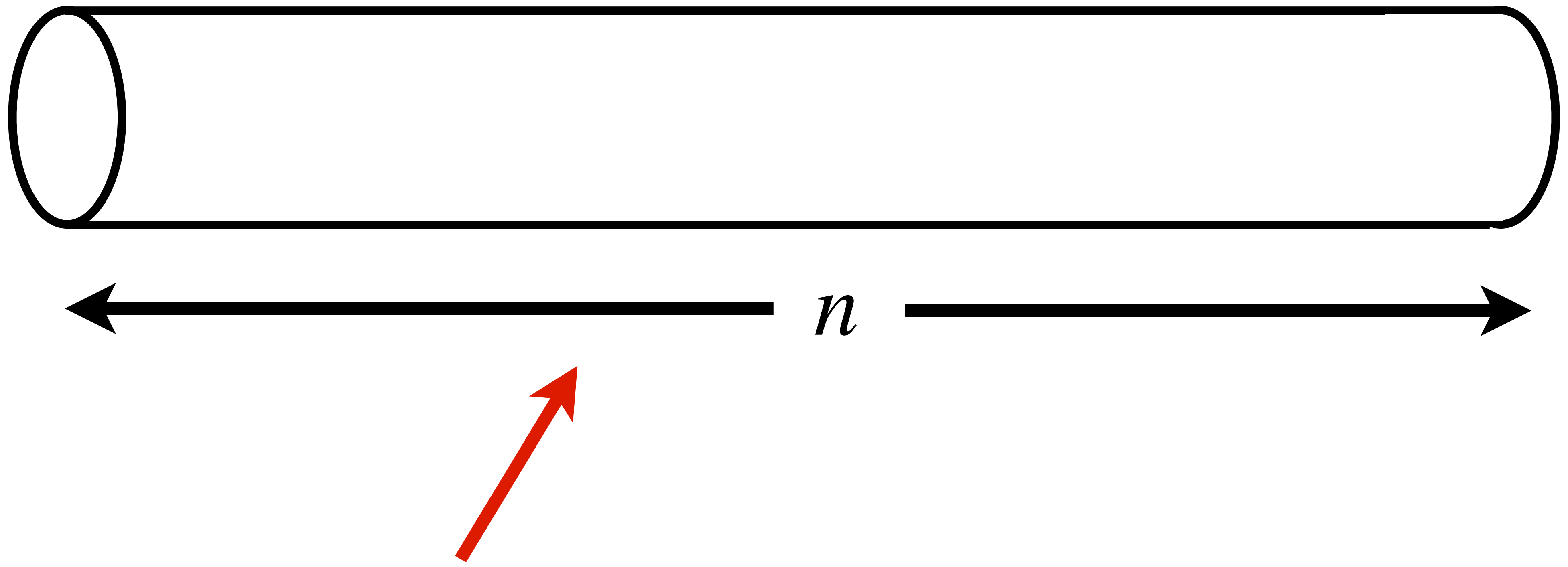

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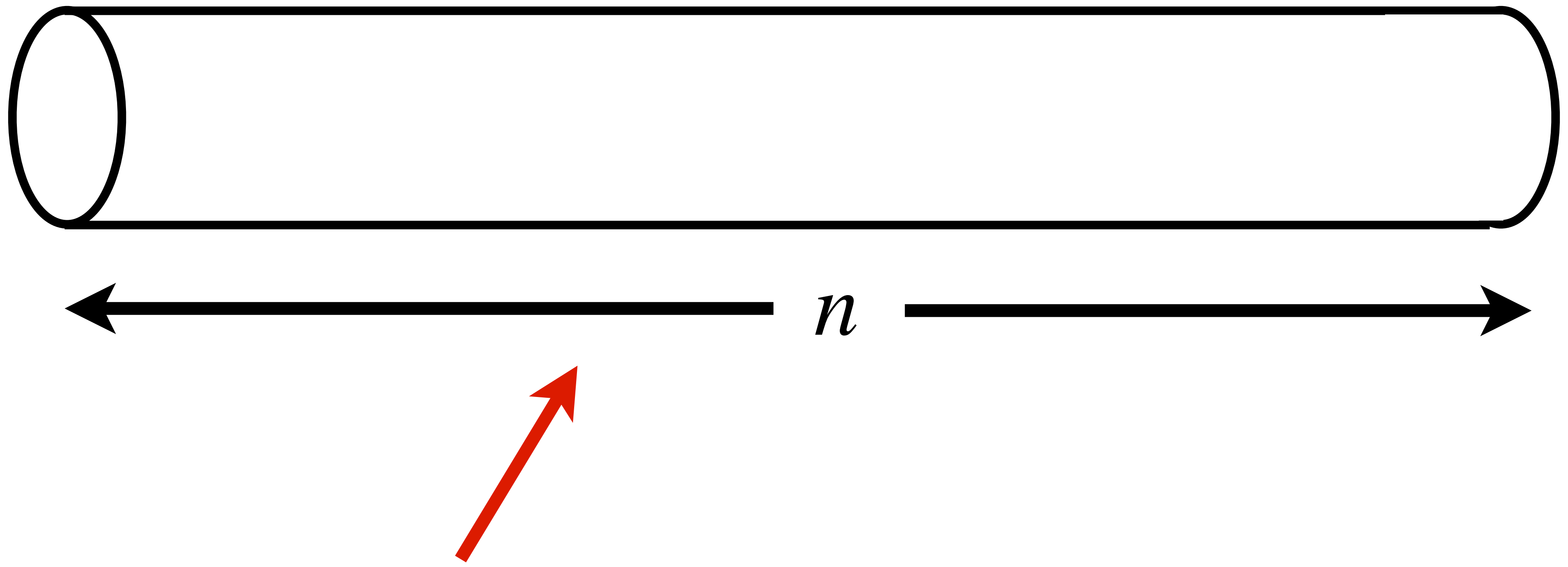

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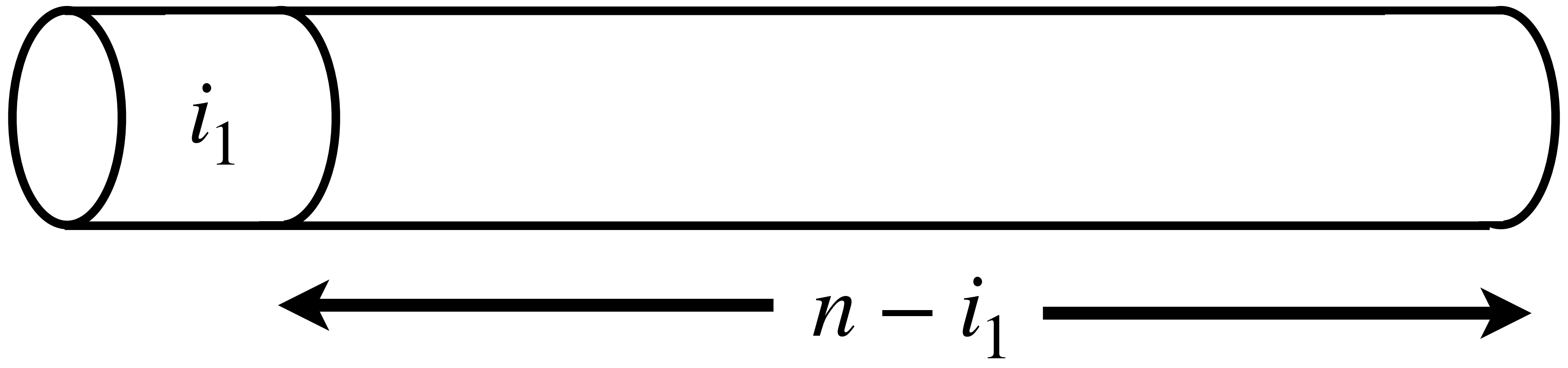

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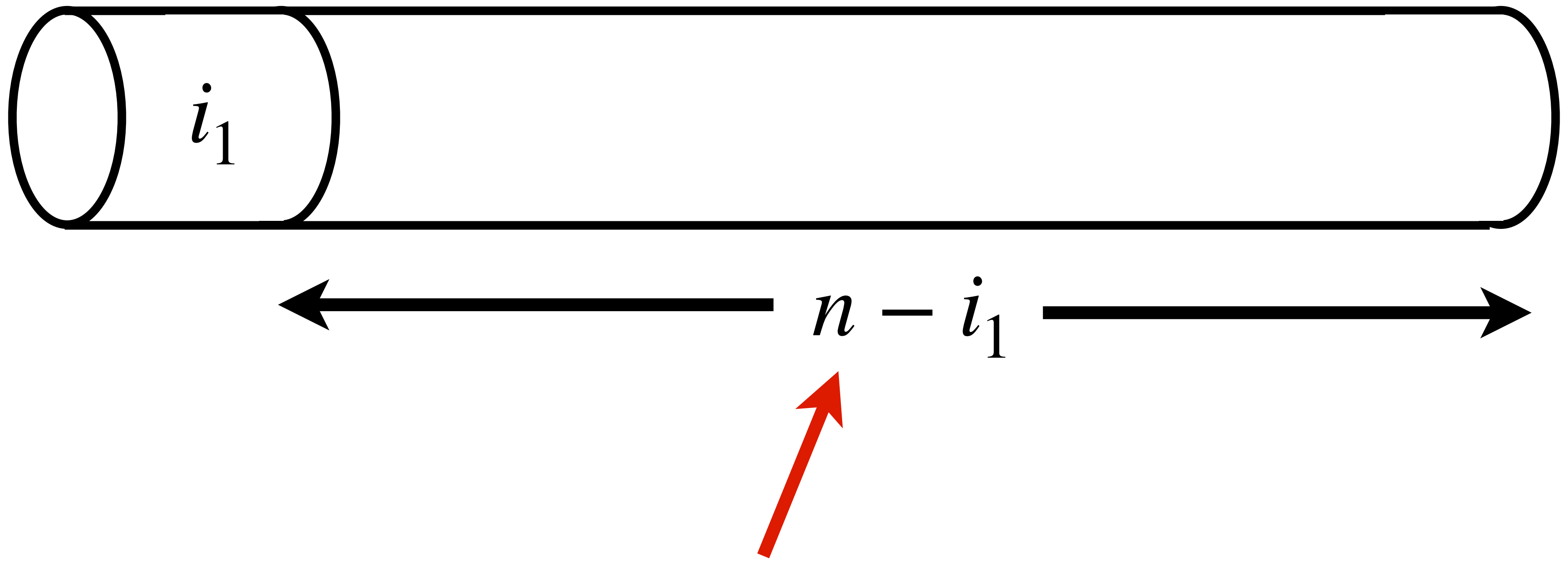

What is the length of the first cut in an $n - i_1$ length rod?

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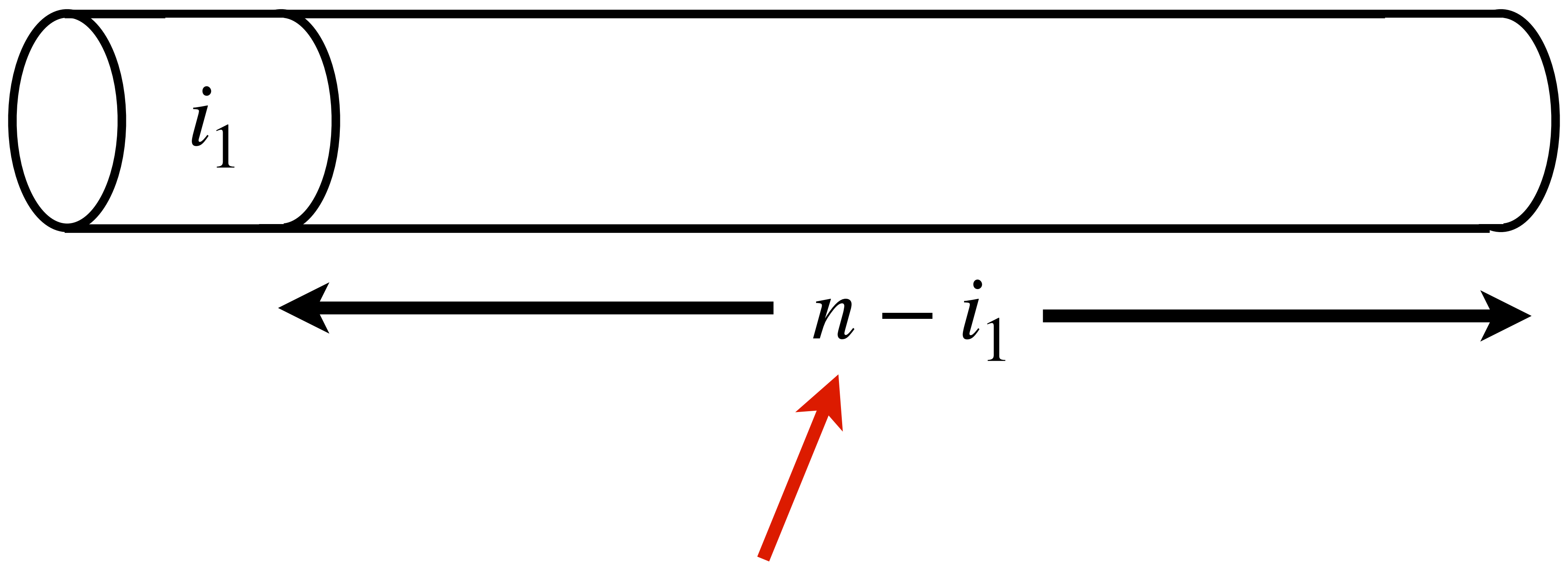

What is the length of the first cut in an $n - i_1$ length rod? i_2.

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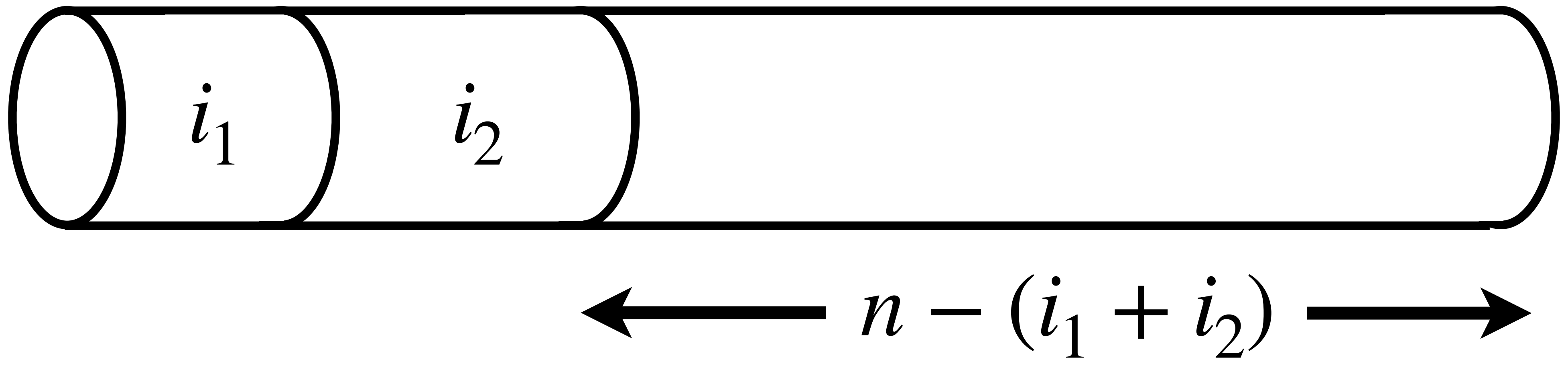

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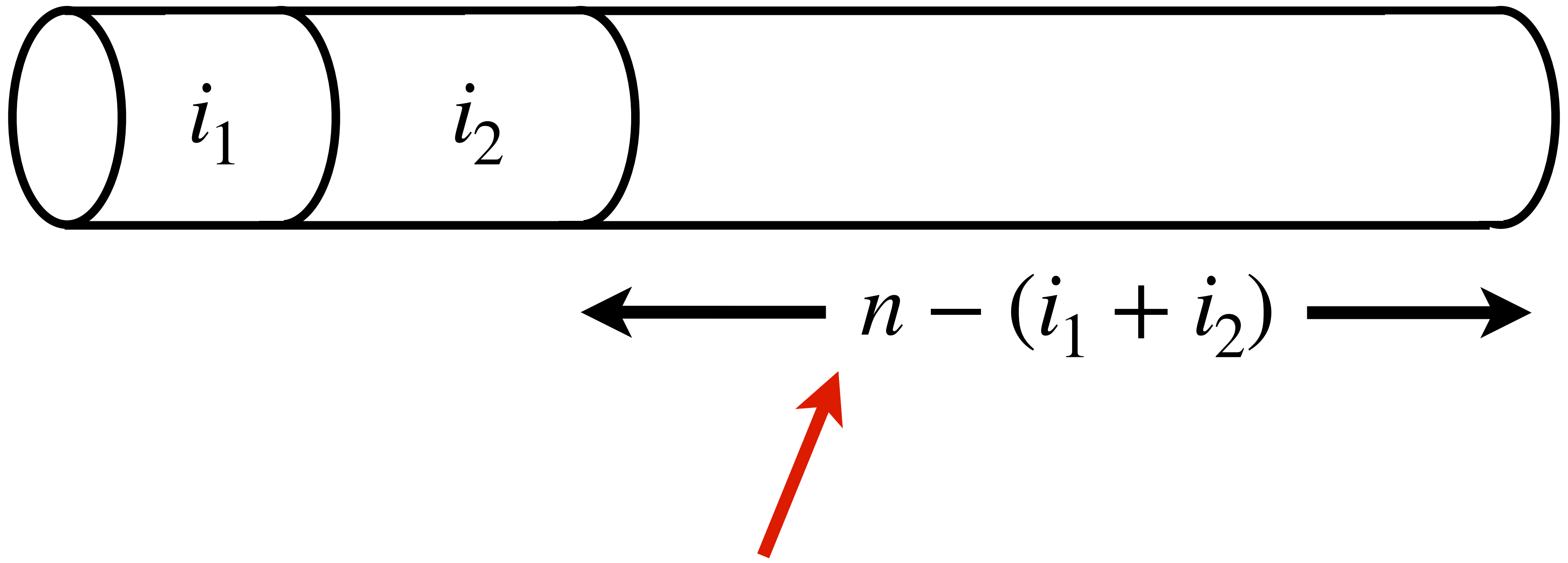

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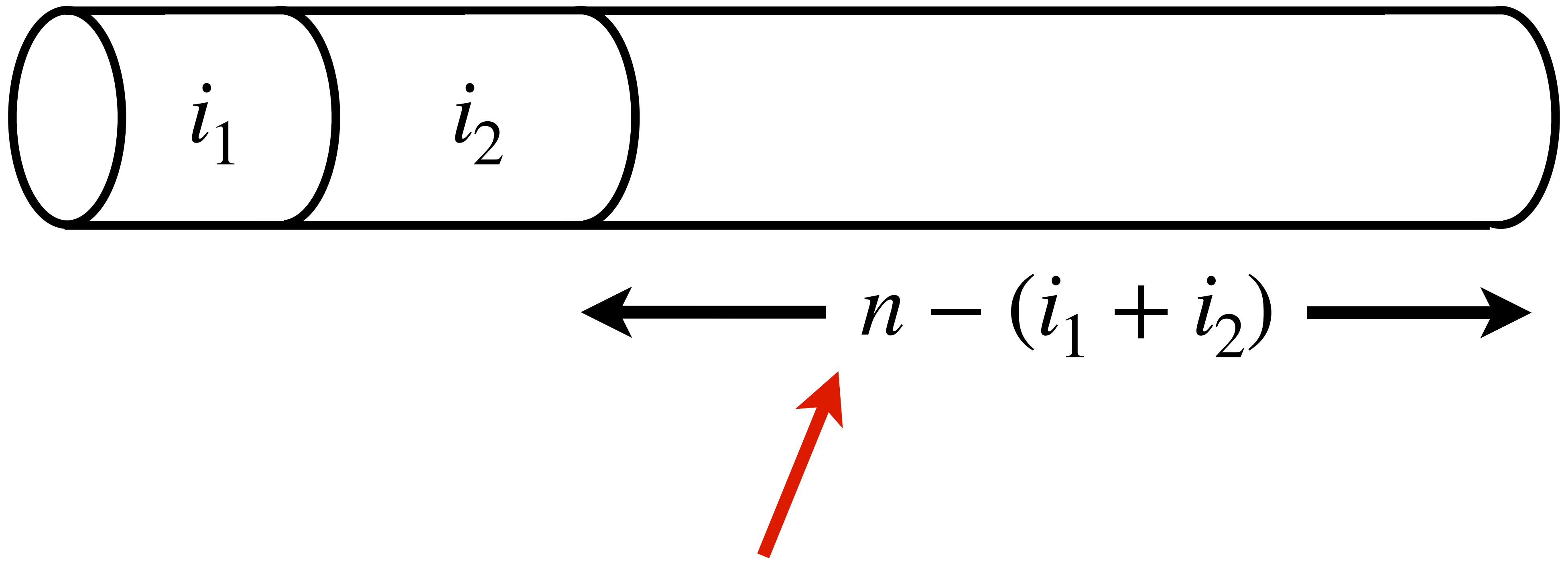

What is the length of the first cut in an $n - (i_1 + i_2)$ length rod? i_3.

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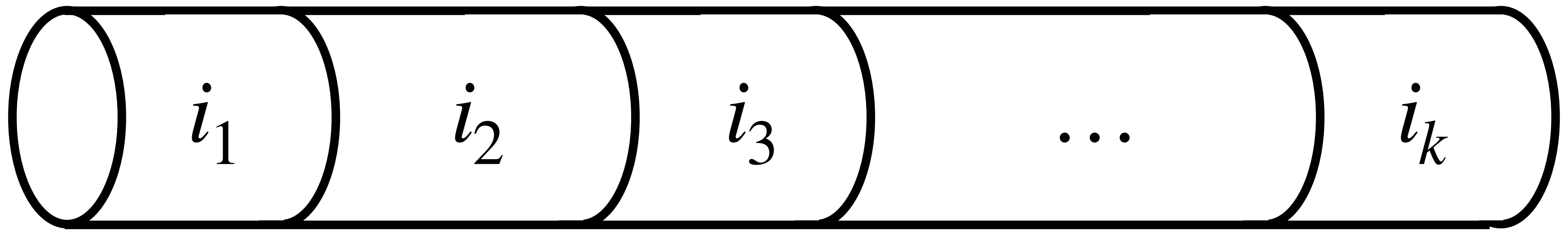

Optimal cutting of an n length rod.

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8. $\quad\quad\quad profit[j] = p[i] + profit[j - i]$
9. $\quad\quad\quad sol[j] =$ ......

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SolutionRC(n, p):
    profit[1 : n] = {p[1],0,…,0}
    sol[1 : n] = {1,0,…,0}
    for j = 2 to n
        profit[j] = p[j]
        sol[j] = j
        for i = 1 to j − 1
            if profit[j] < p[i] + profit[j − i]
                profit[j] = p[i] + profit[j − i]
                sol[j] = i
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SolutionRC(n, p):

$profit[1:n] = \{p[1], 0, \ldots, 0\}$

$sol[1:n] = \{1, 0, \ldots, 0\}$

for $j = 2$ **to** n

    $profit[j] = p[j]$

    $sol[j] = j$

    for $i = 1$ **to** $j - 1$

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return $profit[n]$

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6.       for i = 1 to j − 1
7.           if profit[j] < p[i] + profit[j − i]
8.               profit[j] = p[i] + profit[j − i]
9.               sol[j] = i
10.  return profit[n]
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PrintRC(n, sol):

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PrintRC(n, sol):

```
while n > 0
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                sol[j] = i
    return profit[n]
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sol[*j*] is the length of the first cut in an optimal cutting of *j* length rod.

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PrintRC(n, sol):
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SolutionRC(n, p):

- $\mathit{profit}[1:n] = \{p[1], 0, \ldots, 0\}$
- $\mathit{sol}[1:n] = \{1, 0, \ldots, 0\}$
- **for** $j = 2$ **to** n
  - $\mathit{profit}[j] = p[j]$
  - $\mathit{sol}[j] = j$
  - **for** $i = 1$ **to** $j - 1$
    - **if** $\mathit{profit}[j] < p[i] + \mathit{profit}[j-i]$
      - $\mathit{profit}[j] = p[i] + \mathit{profit}[j-i]$
      - $\mathit{sol}[j] = i$
- **return** $\mathit{profit}[n]$

$\mathit{sol}[j]$ is the length of the first cut in an optimal cutting of j length rod.

PrintRC(n, sol):

- **while** $n > 0$
  - **print** $\mathit{sol}[n]$
  - $n = n - \mathit{sol}[n]$

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Example: Let $S =$ "michaelscott" be a sequence.

Some subsequences of S are : "mcheco", "m", "iaeo", "michaelscott", "".

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Input: Two sequences X and Y.

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Example: $X =$ "iitjodhpur", $Y =$ "iitindore"

Some common subsequences of X and Y are: "iit", "dr", "tor".

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Some longest common subsequences of X and Y are: "iitor", "iitdr".

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- LCS of two sequences or strings is a measure of how similar they are.

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- Used to find similarity of DNAs which can be seen as strings of "A", "C", "G", and "T"

Longest Common Subsequence

LCS:

Input: Two sequences X and Y.

Output: Length of the longest common subsequence.

Application of LCS:

- LCS of two sequences or strings is a measure of how similar they are.
- Used to find similarity of DNAs which can be seen as strings of "A", "C", "G", and "T" characters which represent nucleotides.

Brute Force for LCS

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Generating all the subsequences takes $O(2^{|x|})$ time.

Finding Optimal Substructure in LCS

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What should be the remaining LCS(X, Y)?

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What should be the remaining LCS(X, Y)?

Intuition says it should be LCS of "Thiruvananthapura~~m~~" and "Visakhapatna~~m~~".

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Claim: Let $X = x_1x_2...x_m$ and $Y = y_1y_2...y_n$ be two sequences such that $x_m = y_n$. Then,

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Ending characters are different.

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